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# Optimization and Calculation of Gas Thermal Diffusion Column

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## Abstract

Analytical dependencies, permitting the optimization of a gas thermal diffusion column (TDC), are obtained. A method for calculation of the geometry and gas flows in TDC or a cascade of TDC (TD-cascade) has been proposed.



## 1. INTRODUCTION

The problems of optimization and calculation of gas TDC or TD-cascade used for isotope separation has been thrashed out by Rozen (*1*). The coefficient of separation in isotope separation is low, and cascades rather than a single TDC have to be used. That is why, in Ref. *1*, the optimization of the cascade is considered in detail, but the question about the optimization of separate columns (stages) in the cascade has been described only briefly. Moreover, the obtained dependencies in Ref. *1* are for low values of the coefficient of separation $q$ (or the coefficient of enrichment $\epsilon$), as well as for the thermal diffusion factor $\alpha_T$, and cannot be used for the calculation of TDC or TD-cascade separating gas mixtures with high values of the thermal diffusion factor $\alpha_T$ and $q$ (or $\epsilon$).

The present work is a more general examination of the problems of optimization and calculation of TDC of arbitrary values of the thermal diffusion factor $\alpha_T$ and $q$ and $\epsilon$. It is a continuation of our previous works (*2*, *3*).

In this work, as well as in Ref. *3*, TDC (TD-cascade) is assumed to consist of two columns (cascades), called upper and lower, separated by the leading point in the initial gas mixture. In the upper column (cascade), the light gas concentrates, while the heavy gas concentrates in the lower. The derivation of equations for both columns (cascades) will be made in parallel: on the left-hand side for the lower column and on the right-hand side for the upper column. Common expressions will be centered. We will use the concentrations of the light gas in the derivations for the upper column (cascade) and the concentrations of the heavy gas in the derivations for the lower column (cascade).

## 2. GEOMETRY OF TDC

### 2.1. The Height of TDC

The height of TDC under dynamic conditions, with extraction of product, can be obtained by using the formula proposed by Rozen (*1*):

$$h = Nh_0 \tag{1}$$

where, as shown in Ref. *2*,

$$h_0 = 0.5k_p k_0 \delta_0 \tag{2}$$

$$k_0 = (5/7)^{1/2}(\bar{\delta}^4 + 2\bar{\delta}^{-2}) \tag{3}$$

$$\bar{\delta} = \delta/\delta_0 \tag{4}$$

$$\delta_0 = [(9!/2)^{1/2}\eta D/\rho g \beta \Delta T]^{1/3} \tag{5}$$

$$\Delta T = (T_c + T_H)/2 \tag{6}$$

Usually $k_p = 1.0$–$1.2$ (see Ref. *2*).

The determination of $N$ can be done with the method used in Ref. *1*.

According to Refs. *1* and *3*, the degree of separation in TDC in coordinates $X$–$Y$ is given by the expression

$$Q^* = Q_H^* Q_L^* = [x_k(1 - x_0)]/x_0(1 - x_k) = \epsilon^*(N + \ln Q_L^*) \tag{7}$$

where

$$\epsilon^* = \mu\epsilon \tag{8}$$

$$\mu = (x_2 - x_1)/(1 - \epsilon x_1) \qquad \mu = (x_2 - x_1)/[1 - \epsilon(1 - x_2)] \tag{9}$$

$$Y = (y - y_1)/(y_2 - y_1) \tag{10}$$

$$X = (x - x_1)/(x_2 - x_1) \tag{11}$$

$$Q_H^* = \frac{X_k}{X_0} = \frac{x_k - x_1}{x_0 - x_1} \qquad Q_H^* = \frac{1 - X_0}{1 - X_k} = \frac{x_2 - x_0}{x_2 - x_k} \tag{12}$$

$$Q_L^* = \frac{1 - X_0}{1 - X_k} = \frac{x_2 - x_0}{x_2 - x_k} \qquad Q_L^* = \frac{X_k}{X_0} = \frac{x_k - x_1}{x_0 - x_1} \tag{13}$$

For the upper column and each column from the upper cascade, the inlet is at the bottom of the column and the outlet is at the top of the column. For the lower column and each column from the lower cascade, the picture is just the opposite (see Figs. 1 and 2).

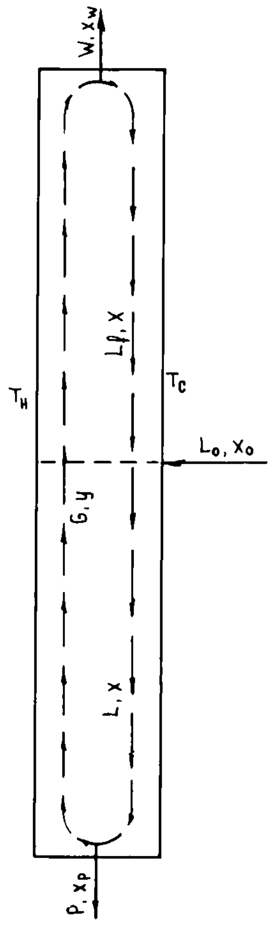


FIG. 1. Scheme of gas flows in a single TDC.

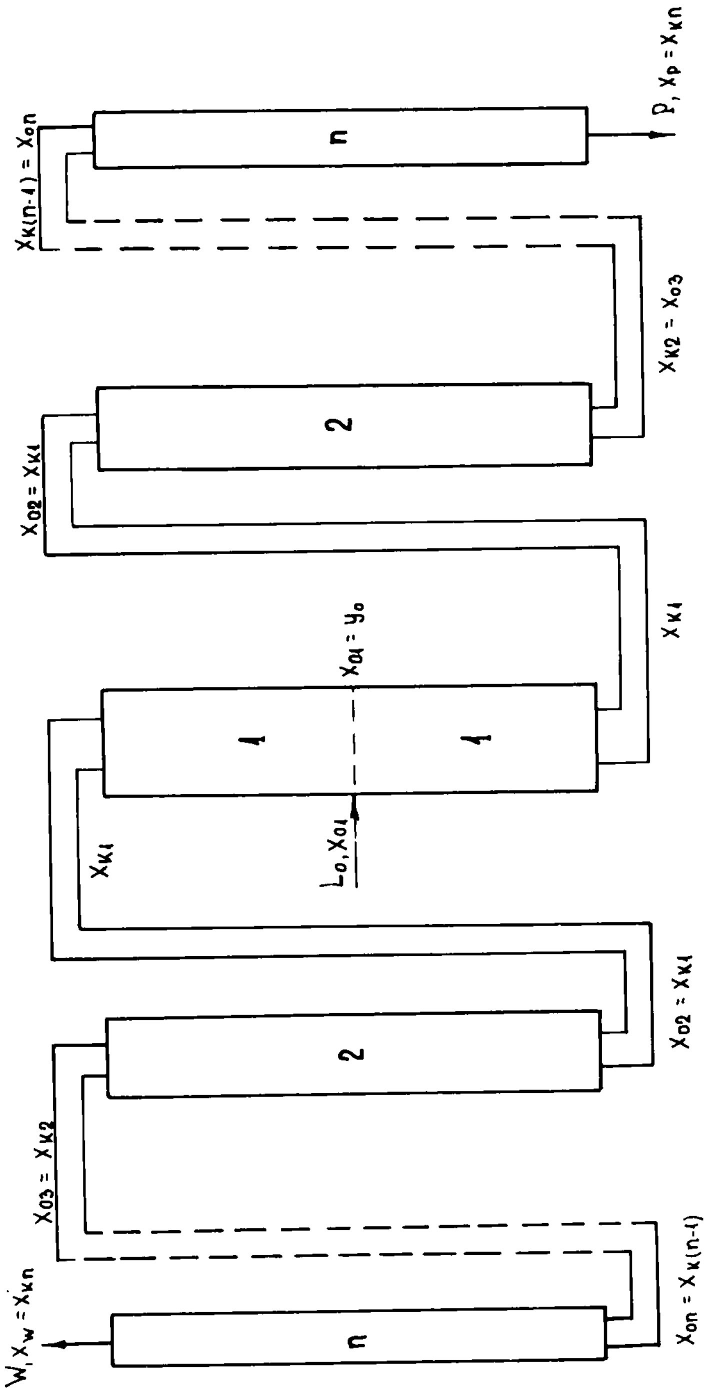


FIG. 2. Scheme of gas flows in a TD-cascade.

From Eq. (7), taking into account Eqs. (8)–(13), for $N$ one obtains

$$N = \frac{1}{\epsilon(x_2 - x_1)}\left[(1 - \epsilon x_1)\ln\frac{x_k - x_1}{x_0 - x_1} + (1 - \epsilon x_2)\ln\frac{x_2 - x_0}{x_2 - x_k}\right]$$

$$N = \frac{1}{\epsilon(x_2 - x_1)}\left\{[1 - \epsilon(1 - x_1)] + \ln\frac{x_k - x_1}{x_0 - x_1} + [1 - \epsilon(1 - x_2)] + \ln\frac{x_2 - x_0}{x_1 - x_k}\right\} \tag{14}$$

and in the general case (see Ref. *3*)

$$x_1 \cong \theta x_0 \tag{15}$$

$$x_2 \cong 1 + \theta x_0 A^* \tag{16}$$

$$A^* = \frac{(1 - \epsilon)(1 - x_p)}{x_p - x_0[1 - \epsilon(1 - x_p)]} \qquad A^* = \frac{1 - x_w}{[1 - \epsilon(1 - x_0)]x_w - x_0} \tag{17}$$

For the four special cases considered in Ref. *3*, Eq. (14) is considerably simplified.

1) Low degrees of separation and concentration of the extraction component (for example, in the last stages of TD-cascade for initial concentrating of isotopes).

$$x_{0i}/x_p > 0.1; \quad x_p < 0.1 \qquad\qquad x_{0i}/x_w > 0.1; \quad x_w < 0.1$$

Subscript $i$ indicates that the corresponding expression relates to the $i$th stage of the TD-cascade. For a single column, $i = 1$ and is therefore omitted.

The degree of separation in TDC in coordinates $x$–$y$ is defined as (see Refs. *1–3*)

$$Q_i = Q_{Hi}Q_{Li} = [x_{ki}(1 - x_{0i})]/x_{0i}(1 - x_{ki}) \tag{18}$$

$$Q_{Hi} = x_{ki}/x_{0i} \qquad\qquad Q_{Hi} = (1 - x_{0i})/(1 - x_{ki}) \tag{19}$$

$$Q_{Li} = (1 - x_{0i})/(1 - x_{ki}) \qquad\qquad Q_{Li} = x_{ki}/x_{0i} \tag{20}$$

Having in mind that in this case $x << 1$ and rendering Eqs. (15)–(20), it can be shown that

$$x_{2i} - x_{1i} \cong x_{2i}$$

$$(x_{2i} - x_{0i})/(x_{2i} - x_{ki}) \cong x_{2i}/x_{2i} = 1.0$$

$$Q_{Li} = (1 - x_{0i})/(1 - x_{ki}) \cong 1.0 \qquad Q_{Hi} = (1 - x_{0i})/(1 - x_{ki}) \cong 1.0$$

$$Q_{Hi} \cong Q_i \qquad Q_{Li} \cong Q_i$$

and

$$N_i \cong \frac{1}{\epsilon_i x_{2i}} \ln \frac{Q_i - \theta_i}{1 - \theta_i} x_{2i} \qquad N_i \cong \frac{1}{(q_i - 1)x_{2i}} \ln \frac{Q_i - \theta_i}{1 - \theta_i} x_{2i}$$

If the coefficient of separation $q_i < 1.1$ ($\epsilon_i < 0.1$), with some increase of $N_i$ obtained (up to 10%), the simpler expression can be used:

$$N_i \cong (1/\epsilon_i) \ln [(Q_i - \theta_i)/(1 - \theta_i)]$$

2) Low degrees of separation and moderate concentrations of the final products

$$x_{0i}/x_p > 0.1;\ 0.1 \leq x_p \leq 0.95 \qquad x_{0i}/x_w > 0.1;\ 0.1 \leq x_w \leq 0.95$$

In this case, taking into account Eqs. (15)–(18), it can be accepted that

$$N_i \cong \frac{1}{\epsilon_i(x_{2i} - x_{1i})} \ln \left[\frac{(Q_{Hi} - \theta_i)(x_{2i} - x_{0i})}{(1 - \theta_i)(x_{2i} - x_{ki})}\right]$$

$$N_i \cong \frac{1}{\epsilon_i(x_{2i} - x_{1i})} \ln \left[\frac{(Q_{Li} - \theta_i)(x_{2i} - x_{0i})}{(1 - \theta_i)(x_{2i} - x_{ki})}\right]$$

with some increase of the $N_i$ obtained, which only for $\epsilon_i > 0.2$ or $Q_{Li} >> Q_{Hi}$ can be over 10%.

3) High degrees of separation and arbitrary concentration of final products.

$$x_{0i}/x_p < 0.1 \qquad x_{0i}/x_w < 0.1$$

Taking into account that $x_{0i} << 1.0$, as well as Eqs. (15)–(18), for $N_i$ one obtains:

$$N_i \cong \frac{1}{\epsilon_i} \ln \frac{Q_{Hi} - \theta_i}{1 - \theta_i} Q_{Li} \qquad N_i \cong \frac{1}{q_i - 1} \ln \frac{Q_{Li} - \theta_i}{1 - \theta_i} Q_{Hi}$$

With an increase of $N_i$ of a few percent, only for $\epsilon_i > 0.2$ or $Q_{Li} >> Q_{Hi}$ can it exceed 10%, and then the more accurate expressions can be used:

$$N_i \cong \frac{1}{\epsilon_i} \ln \frac{Q_{Hi} - \theta_i}{1 - \theta_i} Q_{Li}^{1-\epsilon_i} \qquad N_i \cong \frac{1}{q_i - 1} \ln \left(\frac{Q_{Li} - \theta_i}{1 - \theta_i}\right)^{1-\epsilon_i} Q_{Hi}$$

4) High concentration of the final products.

$$x_p > 0.95 \qquad x_w > 0.95$$

In this situation, when the above inequalities take place, it can be accepted that

$$x_p \approx 1 \qquad x_w \approx 1$$

and with Eq. (14) are transformed to:

$$N_i \cong \frac{1}{\epsilon_i(1 - \theta_i x_{0i})} \ln \frac{Q_{Hi} - \theta_i}{1 - \theta_i} Q_{Li}$$

$$N_i \cong \frac{1}{\epsilon_i(1 - \theta_i x_{0i})} \ln \frac{Q_{Li} - \theta_i}{1 - \theta_i} Q_{Hi}$$

and if $\epsilon > 0.2$ or $Q_{Li} >> Q_{Hi}$:

$$N_i \cong \frac{1}{\epsilon_i(1 - \theta_i x_{0i})} \ln \frac{Q_{Hi} - \theta_i}{1 - \theta_i} Q_{Li}^{1-\epsilon_i}$$

$$N_i \cong \frac{1}{\epsilon_i(1 - \theta_i x_{0i})} \ln \left(\frac{Q_{Li} - \theta_i}{1 - \theta_i}\right)^{1-\epsilon_i} Q_{Hi}$$

## 2.2. Cross Section of TDC and Number of TDCs Working in Parallel

The cross section $S$ of TDC can be presented in two ways. On the one hand

$$S = \frac{\pi(d_1^2 - d_2^2)}{4}n = \pi\frac{d_1 + d_2}{2} \cdot \frac{d_1 - d_2}{2}n = B\delta n = B\bar{\delta}\delta_0 n \qquad (21)$$

since $B = \pi(d_1 + d_2)/2$ and $\delta = \bar{\delta}\delta_0 = (d_1 - d_2)/2$. On the other hand, assuming that $\bar{w}$ = constant from the equation of continuity, it follows that

$$S = (L + G)/\bar{w} \qquad S = (L_f + G)/\bar{w} \qquad (22)$$

Expression (21) is for TDC with cylindrical geometry, since TDC with flat parallel geometry is not practical and will not be examined in the present work.

The average conventional velocity is defined by the following expression (*2*):

$$\bar{w} = 2(7/5)^{1/2}D\bar{\delta}/\delta_0 = 5.6^{0.5}D\bar{\delta}/\delta_0 \qquad (23)$$

and the convectional flows in the general case are defined (*3*) by

$$L_i = P\frac{x_p - x_{0i}[1 - \epsilon_i(1 - x_p)]}{\theta_i\epsilon_i x_{0i}(1 - x_{0i})}$$

$$L_{fi} = W\frac{[1 - \epsilon_i(1 - x_{0i})]x_w - x_{0i}}{\theta_i\epsilon_i x_{0i}(1 - x_{0i})} \qquad (24)$$

$$G = P\frac{(x_p - x_{0i})(1 - \epsilon_i x_{0i})]}{\theta_i\epsilon_i x_{0i}(1 - x_{0i})}$$

$$G = W\frac{(x_w - x_{0i})[1 - \epsilon_i(1 - x_{0i})]}{\theta_i\epsilon_i x_{0i}(1 - x_{0i})} \qquad (25)$$

From Eqs. (21)–(23), $B_i$ is obtained:

$$B_i = \frac{L_i + G_i}{\bar{w}_i\bar{\delta}_i\delta_{0i}} = \frac{L_i + G_i}{5.6^{0.5}D_i\bar{\delta}_i^3} \qquad B_i = \frac{L_{fi} + G_i}{\bar{w}_i\bar{\delta}_i\delta_{0i}} = \frac{L_{fi} + G_i}{5.6^{0.5}D_i\bar{\delta}_i^3} \qquad (26)$$

and taking into account Eqs. (23) and (25):

$$B_i = P\frac{(x_p - x_{0i})(2 - \epsilon_i x_{0i}) + \epsilon_i x_{0i}(1 - x_p)}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}(1 - x_{0i})}$$

$$B_i = W\frac{(x_w - x_{0i})[2 - \epsilon_i(1 - x_{0i})] - \epsilon_i x_w(1 - x_{0i})}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}(1 - x_{0i})} \quad (27)$$

For the case when $\epsilon << 1.0$:

$$B_i = P\frac{(x_p - x_{0i})}{1.4^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}(1 - x_{0i})} \qquad B_i = W\frac{(x_w - x_{0i})}{1.4^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}(1 - x_{0i})}$$

For the above cases, Eq. (27) is modified as follows:

1) $x_{0i}/x_p > 0.1;\ x_p < 0.1 \qquad x_{0i}/x_w > 0.1;\ x_w < 0.1$

$$B_i \cong P\frac{2(x_p - x_{0i}) + \epsilon_i x_{0i}}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}} \qquad B_i \cong W\frac{(2 - \epsilon_i)(x_w - x_{0i}) - \epsilon_i x_w}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}}$$

2) $x_{0i}/x_p > 0.1;\ 0.1 \le x_p \le 0.95 \qquad x_{0i}/x_w > 0.1;\ 0.1 \le x_w \le 0.95$

$$B_i \cong \frac{P(x_p - x_{0i})}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}(1 - x_{0i})} \qquad B_i \cong \frac{W(2 - \epsilon_i)(x_w - x_{0i})}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}(1 - x_{0i})}$$

3) $x_{0i}/x_p < 0.1 \qquad x_{0i}/x_w < 0.1$

$$B_i \cong \frac{P x_p}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}} \qquad B_i \cong \frac{W(1 - \epsilon_i) x_w}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}}$$

4) $x_p > 0.95 \qquad x_w > 0.95$

$$B_i \cong \frac{P}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}} \qquad B_i \cong \frac{W}{5.6^{0.5} D_i \bar{\delta}_i^3 \theta_i \epsilon_i x_{0i}}$$

From Eq. (27) for given $d_1$ or $d_2$ and calculated with Eqs. (4) and (5), $\delta$ ($\bar{\delta}$ is given), the number of the columns working in parallel, can be determined:

$$n_i = \frac{2B_i}{\pi(d_{1i} + d_{2i})} = \frac{2B_i}{\pi(d_{1i} + \delta_i)} = \frac{2B_i}{\pi(d_{2i} - \delta_i)} \quad (28)$$

Or having been given $n_i$, $d_{1i}$ and $d_{2i}$ can be determined.

## 3. CONSUMPTION OF ENERGY AND OPTIMIZATION OF TDC

### 3.1. Energy Consumption

The energy consumption of TDC is the amount of heat that the gas mixture carries from the hot to the cold wall of the column. The necessary energy for maintaining the temperature $T_C$ of the cold wall is not taken into account. In fact only for $T_C$ = 280–300 K (temperature of surroundings) is the total energy consumption approximately identical with theoretical.

Thus, defined energy consumption is calculated with the well-known formula from heat transfer:

$$E = \lambda Bh\Delta T_E/\delta \tag{29}$$

In this case, the temperature difference is not calculated with Eq. (6) and is equal to

$$\Delta T_E = T_H - T_C$$

If the values of $B$, $h$, and $\delta$ from Eqs. (1), (2), (4), and (27) are substituted in Expression (29), the energy consumption is obtained:

$$E_i = (1/7)(1 + 2/\bar{\delta}_i^6)\frac{k_p k_{Ei}\lambda_i N_i \Delta T_{Ei} P}{D_i\theta_i\epsilon_i x_{0i}}$$

$$E_i = (1/7)(1 + 2/\bar{\delta}_i^6)\frac{k_p k_{Ei}\lambda_i N_i \Delta T_{Ei} W}{D_i\theta_i\epsilon_i x_{0i}} \tag{30}$$

where

$$k_{Ei} = \frac{(x_p - x_{0i})(2 - \epsilon_i x_{0i}) + \epsilon_i x_{0i}(1 - x_p)}{1 - x_{0i}} \approx \frac{2(x_p - x_{0i})}{1 - x_{0i}}$$

$$k_{Ei} = \frac{(x_w - x_{0i})[2 - \epsilon_i(1 - x_{0i})] - \epsilon_i x_w(1 - x_{0i})}{1 - x_{0i}} \approx \frac{(2 - \epsilon_i)(x_w - x_{0i})}{1 - x_{0i}}$$

### 3.2. Energetic Optimization of TDC

#### *3.2.1. Optimal Relative Extraction*

From Expression (30) it can be seen that

$$E_i = \text{constant}\ \frac{N_i(\theta_i)}{\theta_i} \tag{31}$$

Hence it follows that for $\theta \to 0$, $E \to \infty$, and as it was shown in Ref. *3*, for $\theta \to 1$, $N \to \infty$, and then also $E_i \to \infty$. Consequently, the energy consumption has limited values and a minimum for $\theta$ in the interval $0 < \theta < 1$.

In order to determine the value of $\theta$, for which the energy consumption is minimum, the minimum of Eq. (31) must be found. For the purpose of simplifying the considerations, the logarithms from Eq. (14) for $N_i$ are expanded as a power series and in the first approximation:

$$\ln \frac{x_{ki} - x_{1i}}{x_{0i} - x_{1i}} \approx \frac{2(x_{ki} - x_{0i})}{x_{0i} + x_{ki} - 2x_{1i}} \tag{32}$$

$$\ln \frac{x_{2i} - x_{0i}}{x_{2i} - x_{ki}} \approx \frac{2(x_{ki} - x_{0i})}{2x_{2i} - (x_{0i} - x_{ki})} \tag{33}$$

Employing Eqs. (14) and (30)–(33):

$$E_i = \frac{a_{1i}a_{4i}}{a_{2i}\theta_i^2 - a_{3i}\theta_i} \tag{34}$$

where

$$a_{1i} = (1/7)(1 + 2/\bar{\delta}_i^6)\frac{k_p k_{Ei} \lambda_i N_i \Delta T_{Ei}}{D_i \epsilon_i x_{0i}} P$$

$$a_{1i} = (1/7)(1 + 2/\bar{\delta}_i^6)\frac{k_p k_{Ei} \lambda_i N_i \Delta T_{Ei}}{D_i \epsilon_i x_{0i}} W$$

$$a_{2i} = \frac{2x_{0i}(1 - x_{0i})[1 - \epsilon_i(1 - x_p)]}{x_p - x_{0i}[1 - \epsilon_i(1 - x_p)]}\left[\frac{2x_p}{1 - \epsilon_i(1 - x_p)} - (x_{0i} - x_{ki})\right]$$

$$a_{2i} = \frac{2x_{0i}(1 - x_{0i})(1 - \epsilon_i x_w)}{[1 - \epsilon_i(1 - x_{0i})]x_w - x_{0i}}\left[\frac{2(1 - \epsilon_i)x_w}{1 - \epsilon_i x_w} - (x_{0i} - x_{ki})\right]$$

$$a_{3i} = (x_{0i} + x_{ki})[2 - (x_{0i} + x_{ki})]$$

$$a_{4i} = [2 - \epsilon_i(x_{0i} - x_{ki})] - a_{5i}$$

$$a_{5i} = 0 \qquad a_{5i} = \frac{2\theta_i \epsilon_i x_{0i}(1 - x_{0i})(1 - \epsilon_i x_w)}{[1 - \epsilon_i(1 - x_{0i})]x_w - x_{0i}}$$

It can be shown that for the upper column and for each column of the upper cascade,

$$a_{5i} << 2 - \epsilon_i(x_{0i} + x_{ki})$$

In this situation with sufficient accuracy it can be assumed that for each case: $a_{1i}$ = constant, $a_{2i}$ = constant, $a_{3i}$ = constant, $a_{4i}$ = constant, $a_{5i} = 0$.

It is not difficult to show that Expression (34) has a maximum for

$$\theta_i = 0.25(Q_{Hi} + 1)k_{\theta i} \qquad \theta_i = 0.25(Q_{Li} + 1)k_{\theta i} \tag{35}$$

where

$$k_{\theta i} = \frac{Q_{Li} + 1}{Q_{Li}\left\{1 + \dfrac{1 - [1 - \epsilon_i(1 - x_p)x_{ki}/x_p]}{1 - [1 - \epsilon_i(1 - x_p)x_{0i}/x_p]}\right\}}$$

$$k_{\theta i} = \frac{Q_{Hi} + 1}{Q_{Li}\left\{1 + \dfrac{1 - [1 - \epsilon_i(1 - x_{ki}) - x_{ki}/x_w]}{1 - [1 - \epsilon_i(1 - x_{0i}) - x_{0i}/x_w]}\right\}} \tag{36}$$

In the case where

$$x_{ki} = x_p \qquad x_{ki} = x_w$$

which is correct for a single column or for the last stage of TD-cascade,

$$k_{\theta i} = \frac{Q_{Li} + 1}{Q_{Li}\left\{1 + \dfrac{\epsilon_i(1 - x_p)}{1 - [1 - \epsilon_i(1 - x_p)x_{0i}/x_p]}\right\}}$$

$$k_{\theta i} = \frac{Q_{Hi} + 1}{Q_{Li}\left\{1 - \dfrac{\epsilon_i(1 - x_w)}{1 - [1 - \epsilon_i(1 - x_{0i}) - x_{0i}/x_w]}\right\}} \tag{37}$$

For the above four cases, the Eqs. (36) and (37) obtained are considerably simplified:

1) $x_{0i}/x_p > 0.1$; $x_p < 0.1$ $\qquad$ $x_{0i}/x_w > 0.1$; $x_w < 0.1$

As shown above, in this case

$$Q_{Li} \cong 1.0 \qquad Q_{Hi} \cong 1.0$$

$$Q_{Hi} \cong Q \qquad Q_{Li} \cong Q$$

when Eq. (36) takes the appearance

$$k_{\theta i} \cong \frac{2(q_i x_p - x_{0i})}{2q_i x_p - (x_{0i} + x_{ki})} \qquad k_{\theta i} \cong \frac{2(x_w - q_i x_{0i})}{2x_w - q_i(x_{0i} + x_{ki})}$$

and Eq. (37)

$$k_{\theta i} \cong \frac{2(q_i x_p - x_{0i})}{2(q_i - 1)x_p - x_{0i}} \qquad k_{\theta i} \cong \frac{2(x_w - q_i x_{0i})}{(2 - q_i)x_w - q_i x_{0i}}$$

2) $x_{0i}/x_p > 0.1;\ 0.1 \leq x_p \leq 0.95$ $\qquad x_{0i}/x_w > 0.1;\ 0.1 \leq x_w \leq 0.95$

$$k_{\theta i} \cong \frac{(Q_{Li} + 1)(x_p - x_{0i})}{Q_{Li}[2 - (x_{0i} + x_{ki})]} \qquad k_{\theta i} \cong \frac{(Q_{Hi} + 1)(x_w - x_{0i})}{Q_{Hi}[2 - (x_{0i} + x_{ki})]}$$

and correspondingly for

$$x_{ki} = x_p \qquad x_{ki} = x_w$$

in the denominator of the above expression, $x_{ki}$ is substituted for with $x_p$ or $x_w$.

3) $x_{0i}/x_p < 0.1$ $\qquad x_{0i}/x_w < 0.1$

In this case, Eq. (36) is reduced to

$$k_{\theta i} \cong \frac{Q_{Li} + 1}{2Q_{Li}} \qquad k_{\theta i} \cong \frac{Q_{Hi} + 1}{2Q_{Hi}}$$

and Eq. (37) to

$$k_{\theta i} \cong \frac{Q_{Li} + 1}{Q_{Li}[1 - \epsilon_i(1 - x_p)]} \qquad k_{\theta i} \cong \frac{Q_{Hi} + 1}{Q_{Hi}[1 - (q_i - 1)(1 - x_w)]}$$

4) $x_p > 0.95$ $\qquad x_w > 0.95$

In all cases

$$k_{\theta i} \cong 1.0$$

When calculating a real column (cascade), one has to keep in mind the circumstance that the minimum of Expression (34) is comparatively flat and allows deviations of the real θ from the optimal one in a comparatively wide region without essentially changing the energy consumption. This is fortuitous for the choice of the real θ, close to the optimal one, in order to have a column with parameters that are more convenient from both constructive and operating points of view. The dependencies obtained are not universal in all cases because of the approximation used. From Eq. (35) it is shown that only for

$$k_{\theta i}(Q_{Hi} + 1) < 4 \qquad k_{\theta i}(Q_{Li} + 1) < 4$$

are the values of $\theta_i$ in the region $0 < \theta_i < 1.0$. This is a result of the fact that the approximation of the logarithms used (see Eqs. 32 and 33) is useful only when

$$(x_{ki} - x_{1i})/(x_{0i} - x_{1i}) < 5.0$$

and

$$(x_{2i} - x_{0i})/(x_{2i} - x_{ki}) < 5.0$$

For higher values the error is considerable. This condition is usually fulfilled in cascades with $Q_i < 5.0$ (see Ref. *1*).

For high degrees of separation, $\theta_{opt}$ must be determined graphically or numerically from Eq. (30).

### 3.2.2. Optimal $\bar{\delta}$

If it is assumed $E = E_0$ for $\bar{\delta} = 1$ and the influence of $\bar{\delta}$ on $k_p$ is neglected, from Eq. (30) it is obtained that for constant $Q$, $T_C$, $T_H$, and pressure $p$ (consequently also $N_i$ and $x_i$):

$$E/E_0 = (1 + 2/\bar{\delta}^6)/3 \qquad (38)$$

It is obvious that for $\bar{\delta} \to \infty$, $E/E_0 \to 1/3$, but even for $\bar{\delta} = 2$, $E/E_0 = 0.343 \approx 1/3$, i.e., for $\bar{\delta} = 2$, $E/E_0 \approx (E/E_0)_{min}$ (see Fig. 3), and for $\bar{\delta} = 1.65$, $E/E_0 = 1.1\ (E/E_0)_{min}$. Consequently from an energetic point of view, $\bar{\delta}_{opt} > 2.0$, but also in the region where $\bar{\delta} = 1.65$–$2.0$, the energy consumption is near to a minimum one (it differs from the theoretical minimum by less than 10%).

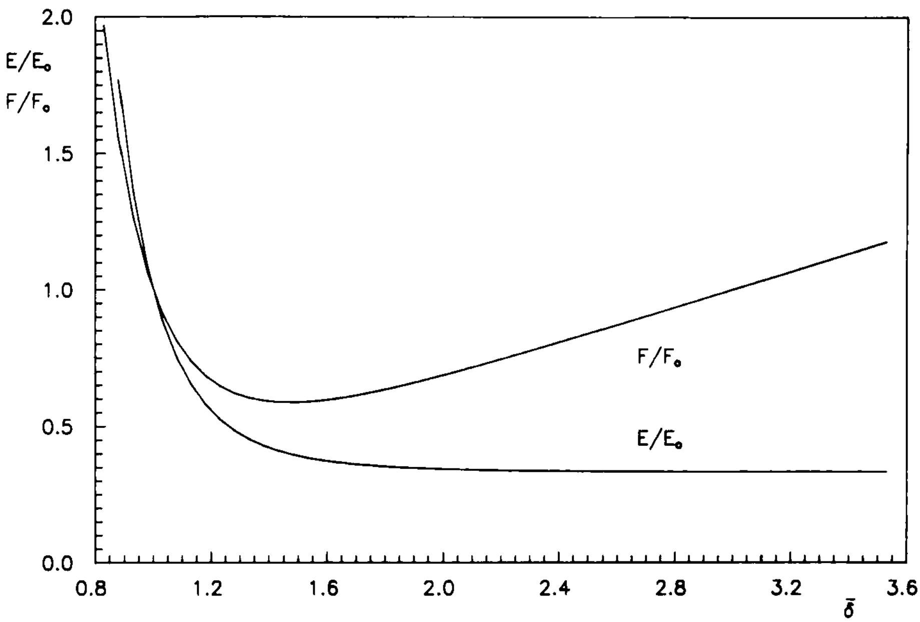


FIG. 3. Relative energy consumption $E/E_0$ and relative space taken by the device $F/F_0$ as a function of the relative distance between the hot and the cold walls of the column.



## 3.3. Instrumental and Gas Dynamic Optimization of TDC

### 3.3.1. Instrumental Optimum

The instrumental optimum is a substantial performance for each real device, because it describes the volume taken up by the apparatus and the consumption of materials for their production. For the thermal diffusion columns the parameter determining the volume taken up by TDC and the consumption of materials is the area $F$, defined as

$$F = Bh = BNh_0 \tag{39}$$

The value of $F$, as seen from Eqs. (14), (27), and (39), depends on $\bar{\delta}$ and $\theta$. If $F = F_0$ is substituted for $\bar{\delta} = 1$ and Eqs. (2), (14), and (21) are taken into account, then from Eq. (39):

$$F/F_0 = (\bar{\delta} + 2/\bar{\delta}^5)/3 \tag{40}$$

It is not difficult to see that Expression (40) has a minimum for $\bar{\delta} = 10^{1/6} = 1.48$. Function (40) is shown in Fig. 3. It is seen that for $\bar{\delta} = 1.22$ to $\bar{\delta} = 1.85$, $F/F_0 \geq 1.1\ (F/F_0)_{min}$ and this is the region of the instrumental optimum.

The instrumental minimum is identical to the energetic one for the relative extraction (productivity of TDC) because from Eqs. (27) and (37) it follows that

$$F \sim N(\theta)/\theta$$

which is analogous to Eq. (31).

### 3.3.2. Gas Dynamic Optimum

It is well-known that for Re > 50 in TDC, a turbulence of the convection gas flows occurs and the thermal diffusion effect is practically zero (*1*, *5*). According to Refs. *1* and *5*, the optimal values of Re are within the region Re = 10–20.

According to Ref. *1*, in TDC

$$\mathrm{Re} = \rho^2 g \delta^3 \Delta T / 360 \eta^2 \overline{T} \quad (41)$$

where

$$\overline{T} = [T_H T_C/(T_H - T_C)] \ln (T_H/T_C)$$

Taking into account Eqs. (4) and (5), Eq. (41) is reduced to

$$\mathrm{Re} = 1.4^{0.5} \overline{\delta}^3 \rho D / \eta \quad (42)$$

From Eq. (42), having in mind that $\mathrm{Re}_{\mathrm{opt}} = 10\text{–}20$, we obtain

$$\overline{\delta}_{\mathrm{opt}} = (1.4^{0.5} \mathrm{Re}_{\mathrm{opt}} \eta / \rho D)^{1/3} \cong (2.05 \div 2.55)(\eta/\rho D)^{1/3} \quad (43)$$

It can be shown from Eq. (43) that $\overline{\delta}_{\mathrm{opt}}$ is independent of the pressure and is practically independent of temperature, which is seen from the shown dependence $\overline{\delta}(\mathrm{Re})$ in Fig. 4 for the He–$N_2$ mixture for arbitrary temperatures and pressures. From the curves shown, $\overline{\delta}$ obviously depends only on the initial concentration $x_0$. Also for Re = 10–20, $\overline{\delta}$ = 1.3–2.5 and the higher values of $\overline{\delta}$ are for higher concentrations of the light gas, in our case helium.

From the above it is concluded that from energetic, instrumental, and gas dynamic points of view, the optimal value of $\overline{\delta}$ is mainly within the region 1.5–2.0. Nevertheless, it is recommended that $\overline{\delta}_{\mathrm{opt}}$ be chosen for each case within the limits defined by Eq. (43), as well as by taking into account the instrumental minimum.

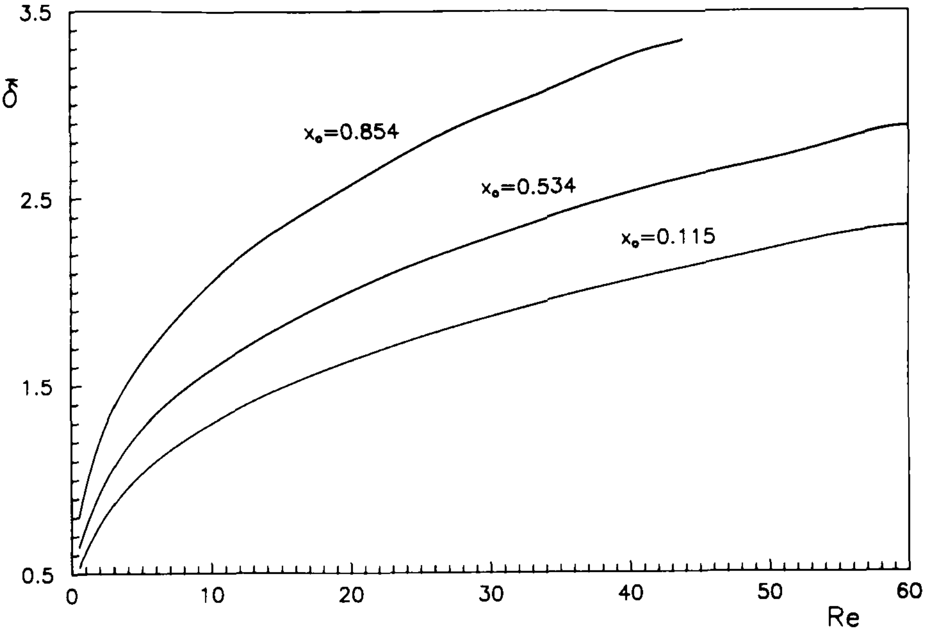


FIG. 4. The criterion Re as a function of relative distance between the hot and the cold walls of the column for He–$N_2$ mixtures at the three initial concentration of helium – $x_0$.



## 4. A MODEL OF CALCULATING THE TDC OR TD-CASCADE

On the basis of the results obtained in the present work, as well as those obtained in our previous works (*2*, *3*), we put forward the following scheme for calculating TDC or TD-cascade.

The optimal temperatures and pressures ($p$,$\overline{T}$,$T_H$,$T_C$) are selected for the corresponding gas mixture on the basis of experimental data (see Ref. *2*) and other considerations.

The degrees of separation $Q_i$ are chosen for each stage of the cascade (for optimal $Q_i$ see Ref. *1*) and $q_i$, $\epsilon_i$, and $\sigma_i$ are calculated (for a single column, $Q$ has been given).

The $\delta_{0i}$ and $\overline{\delta}_{i\text{opt}}(\text{Re})$ are obtained.

The distance between the hot and the cold walls is calculated. If disadvantageous values are obtained for $\delta$, the pressure and $\overline{\delta}$ are changed and the whole procedure is repeated up to this point.

The $\theta_{i\text{opt}}$ are determined and $\theta_i$ are selected.

The $N_i$ are calculated.

The $k_{0i}$ and $h_{0i}$ are calculated, assuming $k_p$ = 1.1–1.2.

The $h_i$ and $B_i$ are calculated.

The number of columns working in parallel, $n_i$, is calculated, and if disadvantageous values for $h_i$ and $n_i$ are obtained, the values of $\theta_i$ or $\bar{\delta}_i$ are changed, as well as eventually $p$, $T_H$, or $T_C$, and all calculations are repeated again.

The $E_i$ and $\Sigma E_i$ are calculated.

This scheme of calculation is convenient for the separation of isotope mixtures and of mixtures of gases with similar properties. If the gases differ from one another in their physical parameters, the model described above can be applied only for very low degrees of separation and for a low concentration of one component. In all other cases the calculation of TDC looks like TD-cascade, because the physical properties of the gas mixture vary substantially along the column height and the calculation of TDC using the properties of the initial gas mixture can lead to large errors. In this case the column is divided in conventional sections with low degrees of separation, specifically in sections in which the concentrations vary slightly and concentration variation can be neglected. The calculation of each section is made by using the shown above scheme for the determination of $h_i$. Further, on the basis of instrumental, energetic, and other considerations, some average values of $B_i$ and $\delta_i$ are chosen to be $B_m$ and $\delta_m$, respectively. Since $\delta_{0i}$ is known for each section, the new values of $\bar{\delta}_i$ are calculated having in mind

$$\delta_i = \delta_m = \bar{\delta}_i \delta_{0i} \tag{44}$$

From Eq. (44):

$$\bar{\delta}_i = \delta_m / \delta_{0i} \tag{45}$$

For each section using Eqs. (41) and (45), $\mathrm{Re}_i$ and $\delta_i$ are calculated, and if their values differ significantly from the optimal values, other values of $\delta_m$ are chosen until adequate results are obtained. Further, the calculations continue in accordance with the above scheme.

## 5. CONCLUSIONS

The analytical dependencies obtained allow complete optimization and calculation of TDC or TD-cascade for the separation of arbitrary binary gas mixtures. The authors hope that their results will be useful for others working in this field.

## SYMBOLS

| | |
|---|---|
| $B$ | average perimeter of TDC (m) |
| $D$ | coefficient of mutual diffusion in gas mixture ($m^2/s$) |
| $d_1$ | diameter of the hot wall of cylindrical TDC (m) |
| $d_2$ | diameter of the cold wall of cylindrical TDC (m) |
| $E$ | energy consumption (W) |
| $F$ | heat exchange area ($m^2$) |
| $G$ | convection flow going up in both columns (cascades) ($Nm^3/s$) |
| $g$ | earth's acceleration ($m/s^2$) |
| $h$ | geometrical height of the column (m) |
| $h_0$ | transfer unit height (TUH) (m) |
| $k_p$ | coefficient taking into account the imperfections of a real column (*2*) |
| $L$, $L_f$ | convection flows going down in lower and upper columns (cascades), respectively ($Nm^3/s$) |
| $N$ | number of TUH on the total height of the column |
| $n$ | number of the columns working in parallel |
| $P$ | product extracted from the bottom of the lower column ($Nm^3/s$) |
| $Q^*$ | degree of separation in TDC, using $X$–$Y$ coordinates |
| $Q_H^*$ | degree of enrichment of the heavy gas in TDC, using $X$–$Y$ coordinates |
| $Q_L^*$ | degree of enrichment of the light gas in TDC, using $X$–$Y$ coordinates |
| $q$ | coefficient of separation |
| $S$ | cross section of working space in TDC ($m^2$) |
| $\Delta T$ | difference between average temperatures of convection flows going up and down (K) |
| $T_H$, $T_C$ | temperatures of the hot and cold walls, respectively (K) |
| $W$ | product extracted from the top of the upper column ($Nm^3/s$) |
| $\overline{w}$ | average velocity of convection gas flows in TDC (m/s) |
| $X$, $Y$ | concentration of extracted gas in the convection flows going down and up, respectively, using $X$–$Y$ coordinates |
| $x$, $y$ | concentration of extracted gas in the convection flows going down and up, respectively |
| $X_0$, $x_0$ | concentration of the extracted gas in the convection flow going down at the inlet of TDC in corresponding coordinates |

$X_k$, $x_k$ — concentration of the extracted gas in the convection flow going down at the outlet of TDC in corresponding coordinates

$x_p$ — concentration of the heavy gas in the product at the bottom of the lower column (the last stage of the lower cascade)

$x_w$ — concentration of the light gas in the product at the top of the upper column (the last stage of the upper cascade)

$x_1$, $y_1$, $x_2$, $y_2$ — coordinates of the two points of intersection of the equilibrium and working lines in coordinates $x$–$y$

## Greek Letters

$\alpha_T$ — thermal diffusion factor

$\beta$ — coefficient of expansion of the gas mixture ($K^{-1}$)

$\delta$ — distance between the hot and the cold walls (m)

$\delta_0$ — optimal distance between the hot and the cold walls (see Refs. *2* or *4*) (m)

$\bar{\delta}$ — ratio of real distance $\delta$ to optimal $\delta_0$

$\epsilon$ — coefficient of enrichment

$\epsilon^*$ — coefficient of enrichment in coordinates $X$–$Y$

$\eta$ — coefficient of dynamical viscosity of a gas mixture (kg/m·s)

$\lambda$ — heat conductivity of separated gas mixture (W/m·K)

$\mu$ — coefficient introduced for short writing of expressions

$\rho$ — density of a gas mixture ($kg/m^3$)

$\theta$ — coefficient of relative extraction (*1*, *3*)

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